

Rank-3 finite temperature logarithmic conformal field theory

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Abstract

We construct a rank-3 finite temperature logarithmic conformal field theory (LCFT) starting from a higher-derivative scalar field model in the BTZ black hole background. Its zero temperature limit reduces to a rank-3 LCFT in the AdS_3 background. For a tricritical generalized massive gravity, we read off the log-square quasinormal frequencies of graviton from the poles of the retarded Green's function in the momentum space. After using the truncation process, we find quasinormal frequencies from a unitary conformal field theory. Finally, employing the retarded Green's functions on the boundary, we compute the absorption cross sections of BTZ black hole which show feature of higher-order differential equations for scalars.

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1 Introduction

Recently, critical gravities have been an actively interesting subject because they were considered as toy models for quantum gravity [1, 2, 3, 4]. At the critical point of avoiding the ghosts, a degeneracy takes place and massive gravitons coincide with massless gravitons. Instead of massive gravitons, an equal amount of logarithmic modes is considered at the critical point. According to the AdS/LCFT correspondence, one finds that a rank-2 logarithmic conformal field theory (LCFT) is dual to a critical gravity [5, 6, 7]. However, one has to resolve the non-unitarity issue of these log-gravity theories. In order to resolve the non-unitarity issue, a polycritical gravity was introduced to provide multiple critical points [8] whose CFT dual is a higher-rank LCFT. On later, it was shown that a consistent unitary truncation of polycritical gravity may be realized at the linearized level for odd rank [9].

The rank of the LCFT refers to the dimensionality of the Jordan cell. Explicitly, the LCFT dual to critical gravity has rank-2 and thus, an operator has its logarithmic partner. The LCFT dual to tricritical gravity has rank-3 and an operator has two logarithmic partners. Most of all, a truncation allows an odd-rank LCFT to be a unitary conformal field theory (CFT) [10]. A six-derivative gravity in three dimensions was treated as dual to a rank-3 LCFT [11], while a four-derivative critical gravity in four dimensions was considered as dual to a rank-3 LCFT [12].

One can construct a rank-3 parity odd theory in the context of four-derivative gravity known as three-dimensional generalized massive gravity (GMG) [13]. There exist two tricritical points in the GMG parameter space whose dual theory is a rank-3 LCFT [14]. At this stage, we would like to point out the following difference in AdS/LCFT correspondence:

- tricritical GMG on the $\text{AdS}_3 \leftrightarrow$ a rank-3 (zero temperature) LCFT
- tricritical GMG on the BTZ black hole [15] \leftrightarrow a rank-3 finite temperature LCFT.

Around the BTZ black hole, the authors [16] have confirmed the AdS/LCFT correspondence between the tricritical GMG and a rank-3 finite temperature LCFT by computing quasinormal frequencies of graviton approximately. In this computation, one could not construct a rank-3 finite temperature LCFT whose zero temperature limits correspond to (3.2)-(3.5). Therefore, one could not make a truncation process to obtain the quasinormal modes from simple poles of CFT.

In this work, we will construct a rank-3 finite temperature LCFT starting from a higher-

derivative scalar field model in the BTZ black hole background. Its zero temperature limit reduces nicely to a rank-3 LCFT (3.2)-(3.5) in the AdS_3 background. We obtain the retarded Green's functions from two-point functions. For a tricritical GMG, we read off the log-square quasinormal frequencies of graviton from the poles of the retarded Green's function in the momentum space. Implementing the truncation process, we find quasinormal frequencies from a unitary finite-temperature CFT. Finally, employing the retarded Green's functions on the boundary, we compute the absorption cross sections of BTZ black hole which involve feature of higher-order differential equations for scalars.

2 Rank-3 LCFT on the BTZ black hole

Let us first consider a gravity action with three scalar fields Φ_1 , Φ_2 , and Φ_3 in three dimensions [10]

$$S = \int d^3x \sqrt{-g} \left[R - 2\Lambda - \partial_\mu \Phi_1 \partial^\mu \Phi_3 - \frac{1}{2}(\partial_\mu \Phi_2)^2 - \Phi_1 \Phi_2 - m^2 \Phi_1 \Phi_3 - \frac{1}{2}m^2 \Phi_2^2 \right]. \quad (2.1)$$

Varying for the fields $g^{\mu\nu}$, Φ_3 , Φ_2 , and Φ_1 on the action (2.1) leads to the equations of motion,

$$\delta g^{\mu\nu}; \quad R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = T_{\mu\nu} \quad (2.2)$$

$$\delta \Phi_3; \quad (\nabla^2 - m^2)\Phi_1 = 0, \quad (2.3)$$

$$\delta \Phi_2; \quad (\nabla^2 - m^2)\Phi_2 = \Phi_1, \quad (2.4)$$

$$\delta \Phi_1; \quad (\nabla^2 - m^2)\Phi_3 = \Phi_2, \quad (2.5)$$

where the energy momentum tensor $T_{\mu\nu}$ is given by

$$T_{\mu\nu} = \partial_\mu \Phi_1 \partial_\nu \Phi_3 + \frac{1}{2} \partial_\mu \Phi_2 \partial_\nu \Phi_2 - \frac{1}{2} g_{\mu\nu} \left(\partial_\rho \Phi_1 \partial^\rho \Phi_3 + \frac{1}{2} (\partial_\rho \Phi_2)^2 + \Phi_1 \Phi_2 + m^2 \Phi_1 \Phi_3 + \frac{1}{2} m^2 \Phi_2^2 \right).$$

We note that varying the action with respect to Φ_3 (Φ_1) leads to the equation for Φ_1 (Φ_3), respectively. This feature will persist in deriving dual CFT operators.

In this work, we introduce a background metric $\bar{g}_{\mu\nu}$ of BTZ black hole with $\Lambda = -1/\ell^2 = -1$ [15],

$$\begin{aligned} ds_B^2 &= \bar{g}_{\mu\nu} dx^\mu dx^\nu \\ &= -\frac{(r^2 - r_+^2)(r^2 - r_-^2)}{r^2} dt^2 + \frac{r^2}{(r^2 - r_+^2)(r^2 - r_-^2)} dr^2 + r^2 \left(d\phi + \frac{r_+ r_-}{r^2} dt \right)^2, \end{aligned} \quad (2.6)$$

being a solution to Eqs. (2.2)-(2.5) together with

$$\bar{\Phi}_i = 0 \quad \text{with } i = 1, 2, 3. \quad (2.7)$$

Here, the ADM mass (M), angular momentum (J), and right/left temperature ($T_{R/L}$) are given by

$$M = r_+^2 - r_-^2, \quad J = 2r_+r_-, \quad T_{R/L} = \frac{r_+ \pm r_-}{2\pi}. \quad (2.8)$$

We consider the perturbation

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad \Phi_i = \bar{\Phi}_i + A_i \quad (2.9)$$

around the BTZ black hole spacetimes. The metric perturbation $h_{\mu\nu}$ has no physical degrees of freedom in three dimensions and thus, we will not consider this further. The relevant quantities are scalar perturbations whose linearized equations are given by

$$(\nabla_B^2 - m^2)A_1 = 0, \quad (2.10)$$

$$(\nabla_B^2 - m^2)A_2 = A_1, \quad (2.11)$$

$$(\nabla_B^2 - m^2)A_3 = A_2, \quad (2.12)$$

which lead to the higher-order equations for A_2 and A_3 as

$$(\nabla_B^2 - m^2)^2 A_2 = 0, \quad (2.13)$$

$$(\nabla_B^2 - m^2)^3 A_3 = 0. \quad (2.14)$$

This means that two scalars A_1 and A_2 are auxiliary fields which are introduced to lower the number of derivatives in the bilinear action $\delta S_{\text{bilinear}}(h_{\mu\nu}, A_i)$. One relevant field is A_3 which satisfies the six-derivative equation (2.14). This perturbative picture for scalar fields A_i is equivalent to ignoring the back reaction of the scalars on the metric [10].

We wish to compute the two-point correlation functions on the boundary and show that they form a rank-3 finite temperature LCFT. The scalars A_i are written in terms of bulk-to-boundary propagators K_{ij} with $i, j = 1, 2, 3$, which relate the bulk solution to the boundary fields $A_{i(b)}$. There are two possibilities [17] to choose the propagators K_{ij} : The first one is the symmetric case where the propagators K_{ij} must be symmetric ($K_{ij} = K_{ji}$) and a bulk-to-boundary propagator for a massive scalar is chosen to be a non-diagonal

propagator [18]. The second one is the Jordan type (or non-symmetric case) [10] where K_{ij} is given by

$$K_{ij} = \begin{pmatrix} K_1 & 0 & 0 \\ K_2 & K_1 & 0 \\ K_3 & K_2 & K_1 \end{pmatrix}. \quad (2.15)$$

Here K_1 ($= K_{11} = K_{22} = K_{33}$), K_2 ($= K_{21} = K_{32}$), and K_3 ($= K_{31}$) satisfy the following relation:

$$(\nabla_B^2 - m^2)K_1 = 0, \quad (2.16)$$

$$(\nabla_B^2 - m^2)K_2 = K_1, \quad (2.17)$$

$$(\nabla_B^2 - m^2)K_3 = K_2. \quad (2.18)$$

In this case, K_1 , K_2 , and K_3 correspond to the bulk-to-boundary propagators of the Klein-Gordon mode (1), log-mode (2), log²-mode (3), respectively for the AdS₃ background.

Upon choosing the Jordan type, the solutions to the Eqs.(2.10)-(2.12) are given by

$$\begin{aligned} A_1(r, u_+, u_-) &= \int du'_+ du'_- \left[K_{11}(r, u_+, u_-; u'_+, u'_-) A_{1(b)} \right] \\ A_2(r, u_+, u_-) &= \int du'_+ du'_- \left[K_{22}(r, u_+, u_-; u'_+, u'_-) A_{2(b)} + K_{21}(r, u_+, u_-; u'_+, u'_-) A_{1(b)} \right] \\ A_3(r, u_+, u_-) &= \int du'_+ du'_- \left[K_{33}(r, u_+, u_-; u'_+, u'_-) A_{3(b)} + K_{32}(r, u_+, u_-; u'_+, u'_-) A_{2(b)} \right. \\ &\quad \left. + K_{31}(r, u_+, u_-; u'_+, u'_-) A_{1(b)} \right], \end{aligned} \quad (2.19)$$

where $u_{\pm} = \phi \pm t$, and $A_{i(b)}$ are functions of (u'_+, u'_-) .

It is well known that in the BTZ black hole background, K_1 can be found as the solution to a homogeneous Klein-Gordon equation (2.16) [19]

$$\begin{aligned} &K_1(r, u_+, u_-; u'_+, u'_-) \\ &= N \left[\frac{\pi^2 T_R T_L}{\frac{r_+^2 - r_-^2}{4r} e^{(\pi T_L \delta u_+ + \pi T_R \delta u_-)} + r \sinh(\pi T_L \delta u_+) \sinh(\pi T_R \delta u_-)} \right]^{\Delta} \\ &\equiv N \left[f(r, u_+, u_-; u'_+, u'_-) \right]^{\Delta}, \end{aligned} \quad (2.20)$$

where $\delta u_{\pm} = u_{\pm} - u'_{\pm}$, $\Delta(\Delta - 2) = m^2$, and the Hawking temperature T_H is defined by

$T_H = 2/(1/T_R + 1/T_L)$. Here N is a normalization constant to be fixed. Using two relations

$$K_2 = \frac{\partial K_1}{\partial m^2} = \frac{1}{2(\Delta - 1)} \frac{\partial K_1}{\partial \Delta}, \quad (2.21)$$

$$K_3 = \frac{1}{2} \frac{\partial K_2}{\partial m^2} = \frac{1}{4(\Delta - 1)} \frac{\partial K_2}{\partial \Delta}, \quad (2.22)$$

K_2 and K_3 can be expressed in terms of K_1 as

$$K_2 = \frac{K_1}{2(\Delta - 1)} \left(\ln[f] + \frac{1}{N} \frac{\partial N}{\partial \Delta} \right), \quad (2.23)$$

$$K_3 = \frac{K_1}{8(\Delta - 1)^2} \left(-2 \frac{K_2}{K_1} + \ln^2[f] + \frac{2}{N} \frac{\partial N}{\partial \Delta} \ln[f] + \frac{1}{N} \frac{\partial^2 N}{\partial \Delta^2} \right). \quad (2.24)$$

We are now in a position to calculate two-point correlation functions from the equations (2.10)-(2.12). For this purpose, we consider an on-shell bilinear action S_{eff} , obtained from surface integral on the boundary s after some integration by parts,

$$S_{\text{eff}}[A_{1(b)}, A_{2(b)}, A_{3(b)}] = - \lim_{r_s \rightarrow \infty} \frac{1}{2} \int_s du_+ du_- \sqrt{-\gamma} \left[A_1(\hat{n} \cdot \nabla) A_3 + A_2(\hat{n} \cdot \nabla) A_2 + A_3(\hat{n} \cdot \nabla) A_1 \right], \quad (2.25)$$

where s is a regulated surface at $r = r_s$, a normal derivative $\hat{n} \cdot \nabla = r \partial_r$, and $\gamma_{\mu\nu}$ is the induced metric on the boundary. We notice that on the boundary, A_i behave as $A_i|_{r \rightarrow \infty} \sim r^{-2+\Delta} A_{i(b)}$. In this case, substituting (2.19) together with (2.20)-(2.24) into the effective action (2.25) leads to

$$\begin{aligned} S_{\text{eff}} = & \frac{\Delta N}{2} \int du_+ du_- du'_+ du'_- \left(\frac{\pi T_L}{\sinh[\pi T_L \delta u_+]} \right)^\Delta \left(\frac{\pi T_R}{\sinh[\pi T_R \delta u_-]} \right)^\Delta \times \\ & \left\{ A_{1(b)}(u) A_{3(b)}(u') + A_{2(b)}(u) A_{2(b)}(u') + A_{3(b)}(u) A_{1(b)}(u') \right. \\ & + \frac{1}{2(\Delta - 1)} \left(\Delta^{-1} + \frac{1}{N} \frac{\partial N}{\partial \Delta} + \ln[\epsilon] + \ln \left[\frac{\pi^2 T_R T_L}{\sinh[\pi T_L \delta u_+] \sinh[\pi T_R \delta u_-]} \right] \right) \times \\ & \left(A_{1(b)}(u) A_{2(b)}(u') + A_{2(b)}(u) A_{1(b)}(u') \right) \\ & + \frac{1}{8(\Delta - 1)^2} \left(\frac{1}{(1 - \Delta) N} \frac{\partial N}{\partial \Delta} + \frac{1}{N} \frac{\partial^2 N}{\partial \Delta^2} + \frac{2}{\Delta} \ln \left[\frac{\pi^2 T_R T_L}{\sinh[\pi T_L \delta u_+] \sinh[\pi T_R \delta u_-]} \right] \epsilon \right. \\ & \left. \left. + \ln^2 \left[\frac{\pi^2 T_R T_L}{\sinh[\pi T_L \delta u_+] \sinh[\pi T_R \delta u_-]} \right] \epsilon^2 \right) A_{1(b)}(u) A_{1(b)}(u') \right\}, \quad (2.26) \end{aligned}$$

where $\epsilon = 1/r_s \rightarrow 0$ as r_s goes infinity.

Following the AdS/LCFT correspondence, we couple the boundary values of the fields to the dual operators as

$$\int d^2x \left[\sum_{i=1}^3 A_{4-i(b)} \mathcal{O}_i \right] \quad (2.27)$$

for the Jordan-type coupling. One can derive the two-point functions for the dual conformal operators \mathcal{O}_i as follows:

$$\langle \mathcal{O}_1(u_+, u_-) \mathcal{O}_1(0) \rangle = \frac{\delta^2 S_{eff}}{\delta A_{3(b)}(u_+, u_-) \delta A_{3(b)}(0)} = 0, \quad (2.28)$$

$$\langle \mathcal{O}_1(u_+, u_-) \mathcal{O}_2(0) \rangle = \frac{\delta^2 S_{eff}}{\delta A_{3(b)}(u_+, u_-) \delta A_{2(b)}(0)} = 0, \quad (2.29)$$

$$\begin{aligned} \xi \langle \mathcal{O}_1(u_+, u_-) \mathcal{O}_3(0) \rangle &= \frac{\delta^2 S_{eff}}{\delta A_{3(b)}(u_+, u_-) \delta A_{1(b)}(0)} \\ &= \Delta N \left(\frac{\pi T_L}{\sinh[\pi T_L u_+]} \right)^\Delta \left(\frac{\pi T_R}{\sinh[\pi T_R u_-]} \right)^\Delta, \end{aligned} \quad (2.30)$$

$$\begin{aligned} \zeta \langle \mathcal{O}_2(u_+, u_-) \mathcal{O}_2(0) \rangle &= \frac{\delta^2 S_{eff}}{\delta A_{2(b)}(u_+, u_-) \delta A_{2(b)}(0)} \\ &= \frac{\Delta N}{2} \left(\frac{\pi T_L}{\sinh[\pi T_L u_+]} \right)^\Delta \left(\frac{\pi T_R}{\sinh[\pi T_R u_-]} \right)^\Delta, \end{aligned} \quad (2.31)$$

$$\begin{aligned} \eta \langle \mathcal{O}_2(u_+, u_-) \mathcal{O}_3(0) \rangle &= \frac{\delta^2 S_{eff}}{\delta A_{2(b)}(u_+, u_-) \delta A_{1(b)}(0)} \\ &= \frac{\Delta N}{2(\Delta - 1)} \left(\frac{\pi T_L}{\sinh[\pi T_L u_+]} \right)^\Delta \left(\frac{\pi T_R}{\sinh[\pi T_R u_-]} \right)^\Delta \times \\ &\quad \left(\frac{1}{\Delta} + \frac{1}{N} \frac{\partial N}{\partial \Delta} + \ln \left[\frac{\pi^2 T_R T_L}{\sinh[\pi T_L u_+] \sinh[\pi T_R u_-]} \right] \epsilon \right), \end{aligned} \quad (2.32)$$

$$\begin{aligned} \chi \langle \mathcal{O}_3(u_+, u_-) \mathcal{O}_3(0) \rangle &= \frac{\delta^2 S_{eff}}{\delta A_{1(b)}(u_+, u_-) \delta A_{1(b)}(0)} \\ &= \frac{\Delta N}{16(\Delta - 1)^2} \left(\frac{\pi T_L}{\sinh[\pi T_L u_+]} \right)^\Delta \left(\frac{\pi T_R}{\sinh[\pi T_R u_-]} \right)^\Delta \times \\ &\quad \left\{ \frac{\Delta}{(1 - \Delta)N} \frac{\partial N}{\partial \Delta} + \frac{\Delta}{N} \frac{\partial^2 N}{\partial \Delta^2} + \frac{2}{\Delta} \ln \left[\frac{\pi^2 T_R T_L}{\sinh[\pi T_L u_+] \sinh[\pi T_R u_-]} \right] \epsilon \right. \\ &\quad \left. + \ln^2 \left[\frac{\pi^2 T_R T_L}{\sinh[\pi T_L u_+] \sinh[\pi T_R u_-]} \right] \epsilon^2 \right\} \end{aligned} \quad (2.33)$$

where arbitrary parameters ξ , ζ , η , and χ will be determined when reducing to the AdS_3 background. This is a rank-3 finite temperature LCFT, our main result.

According to the AdS/CFT dictionary, there are two possibilities of choosing coupling to boundary operators: the first one is symmetric and the second one is Jordan (non-symmetric). The first provides a standard coupling of $\int d^2x [\sum_{i=1}^3 A_{i(b)} \mathcal{O}_i]$, while the second gives us the coupling of $\int d^2x [A_{3(b)} \mathcal{O}_1 + A_{1(b)} \mathcal{O}_3 + A_{2(b)} \mathcal{O}_2]$. Here, we choose the Jordan case which makes the coupling between boundary field $A_{i(b)}$ and operator \mathcal{O}_i less transparent. This implies that in order to obtain the two-point correlator $\langle \mathcal{O}_3 \mathcal{O}_3 \rangle$, one has to vary the effective action (2.26) with respect to $A_{1(b)}$ twice, but $\langle \mathcal{O}_1 \mathcal{O}_1 \rangle$ is obtained when varying with respect to $A_{3(b)}$ twice.

3 Rank-3 LCFT on AdS_3

One can check that in the low temperature limit of $T_{R/L} \rightarrow 0$, two-point functions¹ for $\{\mathcal{O}_1, \mathcal{O}_2, \mathcal{O}_3\}$ reduce to those for $\{\mathcal{O}^{\text{KG}}, \mathcal{O}^{\text{log}}, \mathcal{O}^{\text{log}^2}\}$ defined in the Euclidean Poincaré patch of AdS_3

$$ds_{\text{EAdS}}^2 = \frac{1}{z^2} (dz^2 + dx_1^2 + dx_2^2). \quad (3.1)$$

These are [10]

$$\langle \mathcal{O}^{\text{KG}}(x) \mathcal{O}^{\text{KG}}(0) \rangle = \langle \mathcal{O}^{\text{KG}}(x) \mathcal{O}^{\text{log}}(0) \rangle = 0, \quad (3.2)$$

$$\langle \mathcal{O}^{\text{KG}}(x) \mathcal{O}^{\text{log}^2}(0) \rangle = \langle \mathcal{O}^{\text{log}}(x) \mathcal{O}^{\text{log}}(0) \rangle = \frac{(2(\Delta - 1))^3}{|x|^{2\Delta}}, \quad (3.3)$$

$$\langle \mathcal{O}^{\text{log}}(x) \mathcal{O}^{\text{log}^2}(0) \rangle = \frac{(2(\Delta - 1))^3}{|x|^{2\Delta}} (-2 \ln |x| + \Lambda_1), \quad (3.4)$$

$$\langle \mathcal{O}^{\text{log}^2}(x) \mathcal{O}^{\text{log}^2}(0) \rangle = \frac{(2(\Delta - 1))^3}{|x|^{2\Delta}} (2 \ln^2 |x| + \Lambda_1 \ln |x| + \Lambda_2), \quad (3.5)$$

where $|x| = \sqrt{x_1^2 + x_2^2}$ and constants $\Lambda_{1,2}$ are related to the arbitrary shift parameters $\lambda_{1,2}$, given by the shift transformation: $\phi_i \rightarrow \phi_i + \sum_{k=1}^{i-1} \lambda_k \phi_{i-k}$ ($i = 2, 3$).

¹We mention that log-term in Eq. (2.24) can be eliminated by adding K_2 to K_3 [10]. In this case, the correlation function (2.33) is changed slightly.

From this reduction, N , ξ , ζ , η , χ , Λ_1 , and Λ_2 are determined to be²

$$\xi = \frac{\Delta N}{\left(2(\Delta - 1)\right)^3}, \quad \zeta = \frac{\Delta N}{2\left(2(\Delta - 1)\right)^3}, \quad \eta = \frac{\Delta N}{\left(2(\Delta - 1)\right)^4}, \quad \chi = \frac{\Delta N}{2\left(2(\Delta - 1)\right)^5} \quad (3.6)$$

$$\Lambda_1 = \frac{1}{\Delta} + \frac{1}{N} \frac{\partial N}{\partial \Delta}, \quad \Lambda_2 = \frac{\Delta}{2N} \frac{\partial^2 N}{\partial \Delta^2} - \frac{\Delta}{N^2} \left(\frac{\partial N}{\partial \Delta} \right)^2, \quad (3.7)$$

where $N = c\Delta^{-3}$ with an integration constant c . The connection between u_{\pm} and $x_{1/2}$ is given by

$$u_+ = x_1 + ix_2, \quad u_- = x_1 - ix_2. \quad (3.8)$$

Actually, Eqs.(3.2)-(3.5) correspond to the correlation functions for the un-truncated rank-3 LCFT model. For odd rank, the theory has a unitary subspace which could be obtained by truncating half of the logarithmic modes. An easy way to see how this works for a rank-3 case is first to rewrite the two-point correlation functions schematically as

$$\langle \mathcal{O}^i \mathcal{O}^j \rangle \sim \begin{pmatrix} 0 & 0 & \text{CFT} \\ 0 & \text{CFT} & \text{L} \\ \text{CFT} & \text{L} & \text{L}^2 \end{pmatrix}, \quad (3.9)$$

where $i, j = \text{KG}, \log, \log^2$, CFT denotes the CFT two-point function (3.3), L represents log two-point function (3.4), and L^2 denotes log-square two-point function (3.5). If one truncates the theory to be unitary, one throws away all modes which generates the third column and row of the above matrix. The only non-zero correlation function is given by a reduced matrix as

$$\langle \mathcal{O}^i \mathcal{O}^j \rangle \sim \begin{pmatrix} 0 & 0 \\ 0 & \text{CFT} \end{pmatrix}, \quad (3.10)$$

which implies that the remaining sector involves a non-trivial two-point correlator

$$\langle \mathcal{O}^{\log}(x) \mathcal{O}^{\log}(0) \rangle = \frac{\left[2(\Delta - 1)\right]^3}{|x|^{2\Delta}}. \quad (3.11)$$

This is unitary and thus, the non-unitary issue could be resolved by truncating a rank-3 LCFT.

²Note that considering the shift transformation of $A_i \rightarrow A_i + \sum_{k=1}^{i-1} \tilde{\lambda}_k A_{i-k}$ ($i = 2, 3$) with arbitrary parameters $\tilde{\lambda}$, two parameters Λ_1 and Λ_2 remain undetermined as functions of $(\tilde{\lambda}_1, \tilde{\lambda}_2)$.

4 Retarded Green's functions of rank-3 LCFT

The AdS/LCFT correspondence implies that quasinormal frequencies $\omega = \omega_r - i\omega_i$ determining the relaxation times of perturbations A_i around the BTZ black hole are agreed with the location of poles of the retarded Green's function of the corresponding perturbations \mathcal{O}_i in the dual CFT [20]. The quasinormal modes are obtained by imposing the boundary conditions: ingoing waves near the horizon and the Dirichlet condition at infinity because in asymptotically AdS spaces, the spacelike infinity acts like a reflecting boundary. The regime of near equilibrium real-time evolution can be captured via the linear response theory. A major role is being played by the pole of these holographic response functions (quasinormal frequencies). Actually, the retarded Green's functions are the central objects in linear response theory which are defined by

$$\mathcal{D}_{jk}^{\text{ret}}(t, \phi; 0, 0) = i\Theta(t - 0)\bar{\mathcal{D}}_{jk}(u_+, u_-), \quad j, k = 1, 2, 3, \quad (4.1)$$

where the commutator evaluated in the equilibrium canonical ensemble are given by

$$\bar{\mathcal{D}}_{jk}(u_+, u_-) = \left\langle \mathcal{O}_j(u_+ - i\epsilon, -u_- - i\epsilon)\mathcal{O}_k(0) \right\rangle - \left\langle \mathcal{O}_j(u_+ + i\epsilon, -u_- + i\epsilon)\mathcal{O}_k(0) \right\rangle. \quad (4.2)$$

Making the Fourier transform of $\bar{\mathcal{D}}_{jk}(u_+, u_-)$, the commutator $\bar{\mathcal{D}}_{jk}(p_+, p_-)$ takes the form

$$\bar{\mathcal{D}}_{jk}(p_+, p_-) = \int du_+ du_- e^{i(p_+ u_+ - p_- u_-)} \bar{\mathcal{D}}_{jk}(u_+, u_-) \quad (4.3)$$

in the momentum space. Here $p_{\pm} = (\omega \mp k)/2$. Using (4.3) and (2.28) [(2.29)], one finds that

$$\bar{\mathcal{D}}_{11}(p_+, p_-) = \bar{\mathcal{D}}_{12}(p_+, p_-) = 0. \quad (4.4)$$

Plugging (2.30) [(2.31)] into (4.3), we obtain the commutator $\bar{\mathcal{D}}_{13}(p_+, p_-)$ [$\bar{\mathcal{D}}_{22}(p_+, p_-)$],

$$\begin{aligned} \bar{\mathcal{D}}_{13}(p_+, p_-) &= \bar{\mathcal{D}}_{22}(p_+, p_-) \\ &= \left(2(\Delta - 1)\right)^3 \int du_+ du_- e^{ip_+ u_+} e^{-ip_- u_-} \left\{ \left(\frac{\pi T_L}{\sinh[\pi T_L(u_+ - i\epsilon)]} \right)^\Delta \times \right. \\ &\quad \left. \left(\frac{\pi T_R}{\sinh[\pi T_R(u_- - i\epsilon)]} \right)^\Delta - \left(\frac{\pi T_L}{\sinh[\pi T_L(u_+ + i\epsilon)]} \right)^\Delta \left(\frac{\pi T_R}{\sinh[\pi T_R(u_- + i\epsilon)]} \right)^\Delta \right\} \\ &= \left(2(\Delta - 1)\right)^3 \frac{(2\pi T_L)^{\Delta-1} (2\pi T_R)^{\Delta-1}}{\Gamma(\Delta)^2} \sinh \left[\frac{p_+}{2T_L} + \frac{p_-}{2T_R} \right] \times \\ &\quad \left| \Gamma \left(\frac{\Delta}{2} + i \frac{p_+}{2\pi T_L} \right) \right|^2 \left| \Gamma \left(\frac{\Delta}{2} + i \frac{p_-}{2\pi T_R} \right) \right|^2 \end{aligned} \quad (4.5)$$

where Γ is the gamma function. In deriving this, we have used the formula

$$\int dx e^{-i\omega x} (-1)^\Delta \left(\frac{\pi T}{\sinh[\pi T(x \pm \epsilon)]} \right)^{2\Delta} = \frac{(2\pi T)^{2\Delta-1}}{\Gamma(2\Delta)} e^{\pm i\omega/2T} \left| \Gamma\left(\Delta + i\frac{\omega}{2\pi T}\right) \right|^2. \quad (4.6)$$

At this stage, we wish to point out that it is not easy to compute the commutators $\bar{\mathcal{D}}_{23}(p_+, p_-)$ and $\bar{\mathcal{D}}_{33}(p_+, p_-)$ directly because they have the logarithmic singularities as are shown in (2.32) and (2.33). It is suggested, however, that $\bar{\mathcal{D}}_{23}$ can be deduced from the relation [18]

$$\left\langle \mathcal{O}_2(u_+, u_-) \mathcal{O}_3(0) \right\rangle = \frac{\partial}{\partial \Delta} \left\langle \mathcal{O}_1(u_+, u_-) \mathcal{O}_3(0) \right\rangle. \quad (4.7)$$

We compute $\bar{\mathcal{D}}_{23}(p_+, p_-)$ by using the relation (4.7), which yields

$$\begin{aligned} \bar{\mathcal{D}}_{23}(p_+, p_-) &= \left\{ \frac{3}{\Delta - 1} + \ln[2\pi T_L] + \ln[2\pi T_R] - 2\psi(\Delta) + \frac{1}{2}\psi\left(\frac{\Delta}{2} + i\frac{p_+}{2\pi T_L}\right) \right. \\ &\quad \left. + \frac{1}{2}\psi\left(\frac{\Delta}{2} - i\frac{p_+}{2\pi T_L}\right) + \frac{1}{2}\psi\left(\frac{\Delta}{2} + i\frac{p_-}{2\pi T_R}\right) + \frac{1}{2}\psi\left(\frac{\Delta}{2} - i\frac{p_-}{2\pi T_R}\right) \right\} \bar{\mathcal{D}}_{13}(p_+, p_-) \\ &\equiv \Psi(p_+, p_-) \bar{\mathcal{D}}_{13}(p_+, p_-), \end{aligned} \quad (4.8)$$

where $\psi(A) = \partial \ln[\Gamma(A)]/\partial A$ is the digamma function. Importantly, we may calculate $\bar{\mathcal{D}}_{33}$ when using the relation

$$\left\langle \mathcal{O}_3(u_+, u_-) \mathcal{O}_3(0) \right\rangle = \frac{1}{2} \frac{\partial^2}{\partial \Delta^2} \left\langle \mathcal{O}_1(u_+, u_-) \mathcal{O}_3(0) \right\rangle. \quad (4.9)$$

Accordingly, $\bar{\mathcal{D}}_{33}$ takes the form in the momentum space

$$\begin{aligned} \bar{\mathcal{D}}_{33}(p_+, p_-) &= \left\{ -\frac{3}{2(\Delta - 1)^2} - \psi'(\Delta) + \frac{1}{8}\psi'\left(\frac{\Delta}{2} + i\frac{p_+}{2\pi T_L}\right) + \frac{1}{8}\psi'\left(\frac{\Delta}{2} - i\frac{p_+}{2\pi T_L}\right) \right. \\ &\quad \left. + \frac{1}{8}\psi'\left(\frac{\Delta}{2} + i\frac{p_-}{2\pi T_R}\right) + \frac{1}{8}\psi'\left(\frac{\Delta}{2} - i\frac{p_-}{2\pi T_R}\right) + \frac{1}{2}\Psi(p_+, p_-)^2 \right\} \bar{\mathcal{D}}_{13}(p_+, p_-), \end{aligned} \quad (4.10)$$

where the prime ($'$) denotes the differentiation with respect to Δ .

It is worth noting that for the non-rotating BTZ black hole ($T_R = T_L = T_H = 1/2\pi\ell$, $r_+ = \ell$), the pole structure of the commutators $\bar{\mathcal{D}}_{13}(p_+) = \bar{\mathcal{D}}_{22}(p_+)$, $\bar{\mathcal{D}}_{23}(p_+)$,

and $\bar{\mathcal{D}}_{33}(p_+)$ are given by (after setting $\Delta = 2h_L$)

$$\bar{\mathcal{D}}_{13}(p_+) \propto \Gamma\left(h_L + i\frac{p_+}{2\pi T_L}\right) \Gamma\left(h_L - i\frac{p_+}{2\pi T_L}\right), \quad (4.11)$$

$$\begin{aligned} \bar{\mathcal{D}}_{23}(p_+) \propto & \Gamma'\left(h_L + i\frac{p_+}{2\pi T_L}\right) \Gamma\left(h_L - i\frac{p_+}{2\pi T_L}\right) \\ & + \Gamma\left(h_L + i\frac{p_+}{2\pi T_L}\right) \Gamma'\left(h_L - i\frac{p_+}{2\pi T_L}\right), \end{aligned} \quad (4.12)$$

$$\begin{aligned} \bar{\mathcal{D}}_{33}(p_+) \propto & \Gamma''\left(h_L + i\frac{p_+}{2\pi T_L}\right) \Gamma\left(h_L - i\frac{p_+}{2\pi T_L}\right) \\ & + 2\Gamma'\left(h_L + i\frac{p_+}{2\pi T_L}\right) \Gamma'\left(h_L - i\frac{p_+}{2\pi T_L}\right) \\ & + \Gamma\left(h_L + i\frac{p_+}{2\pi T_L}\right) \Gamma''\left(h_L - i\frac{p_+}{2\pi T_L}\right), \end{aligned} \quad (4.13)$$

which are consistent with those obtained in the literatures [20, 21, 16]. For a tricritical GMG, we read off the log-square quasinormal frequencies of graviton with $h_L = 0$

$$\omega_t = k - i4\pi T_L(n + h_L). \quad (4.14)$$

from a triple pole of the retarded Green's function in the momentum space $\bar{\mathcal{D}}_{33}(p_+)$.

We represent the whole pole structures as a three-by-three matrix

$$\bar{\mathcal{D}}_{ij} \sim \begin{pmatrix} 0 & 0 & \bar{\mathcal{D}}_{13} \\ 0 & \bar{\mathcal{D}}_{22} & \bar{\mathcal{D}}_{23} \\ \bar{\mathcal{D}}_{13} & \bar{\mathcal{D}}_{23} & \bar{\mathcal{D}}_{33} \end{pmatrix}, \quad (4.15)$$

where $\bar{\mathcal{D}}_{13}[= \bar{\mathcal{D}}_{22}]$ represent a simple pole, $\bar{\mathcal{D}}_{23}$ denotes a double pole, and $\bar{\mathcal{D}}_{33}$ represents a triple pole in the lower-half plane. These pole structures are the same as the retarded Green's function (4.1).

What happens for the quasinormal modes when truncating the log-square quasinormal modes on the BTZ black hole [16]? At this stage, we may answer to this question because we did construct all retarded Green's functions. Applying the previous truncation process done for the LCFT to (4.15) leads to

$$\bar{\mathcal{D}}_{ij} \sim \begin{pmatrix} 0 & 0 \\ 0 & \bar{\mathcal{D}}_{22} \end{pmatrix}, \quad (4.16)$$

which provides a simple pole existing in the finite temperature CFT. From (4.11), one reads off quasinormal frequencies as

$$\omega_s = k - i4\pi T_L(n + h_L). \quad (4.17)$$

5 Absorption cross sections

In this section, we wish to calculate the s -wave absorption cross section (or greybody factor) by using the retarded Green's functions in the momentum spaces. Actually, it is not an easy task to derive all absorption cross sections by solving equations of motion directly because two scalars A_2 and A_3 satisfy the higher-order differential equations (2.13) and (2.14) on the BTZ black hole. However, when using the boundary retarded Green's functions, one computes the absorption cross sections easily.

It is well known that the absorption cross section [22, 23, 24] can be written in terms of frequency (ω) and temperature ($T_{R/L}$, T_H):

$$\sigma_{\text{abs}}^{ij} = \frac{\mathcal{C}_0}{\omega} \bar{\mathcal{D}}_{ij}(\omega), \quad (5.1)$$

where \mathcal{C}_0 is a normalization constant. Here $\bar{\mathcal{D}}_{ij}(\omega)$ is obtained by substituting $p_+ = p_- = \omega/2$ for s -wave ($k = 0$) into (4.4), (4.5), (4.8), and (4.10). From the definition (5.1), one can find σ_{abs}^{13} as

$$\begin{aligned} \sigma_{\text{abs}}^{13} = \sigma_{\text{abs}}^{22} = & \frac{\mathcal{C}_0 \left(2(\Delta - 1)\right)^3 (2\pi T_L \ell)^{\Delta-1} (2\pi T_R \ell)^{\Delta-1} \sinh(\frac{\omega}{2T_H})}{\omega \Gamma^2(\Delta)} \\ & \times \left| \Gamma\left(\frac{\Delta}{2} + i\frac{\omega}{4\pi T_L}\right) \Gamma\left(\frac{\Delta}{2} + i\frac{\omega}{4\pi T_R}\right) \right|^2 \end{aligned} \quad (5.2)$$

where the AdS_3 curvature radius $\ell = 1$ is restored for convenience. For $\Delta = 2$ ($m^2 = 0$), σ_{abs}^{13} is the same with the absorption cross section for a massless minimally coupled scalar which satisfies $\nabla_B^2 A_1 = 0$ [25, 26]

$$\sigma_{\text{abs}}^{13} = \sigma_{\text{abs}}^{22} = \pi^2 \omega \ell^2 \frac{e^{\omega/T_H} - 1}{(e^{\omega/2T_L} - 1)(e^{\omega/2T_R} - 1)} \quad (5.3)$$

with $\mathcal{C}_0 = 1/4$. One can easily check that in the low-energy limit of $\omega \ll T_{R/L}$, the absorption cross section σ_{abs}^{13} reduces to the area-law of $\sigma_{\text{abs}}^{13}|_{\omega \ll T_{R/L}} = 2\pi r_+ = \mathcal{A}_H$, while in the low-temperature limit of $\omega \gg T_{R/L}$, it becomes $\sigma_{\text{abs}}^{13}|_{\omega \gg T_{R/L}} = \pi^2 \omega \ell^2$.

On the other hand, from the relation of (4.8), we find the absorption cross section

$$\sigma_{\text{abs}}^{23} \simeq \left[\frac{3}{\Delta - 1} + \ln(2\pi T_L \ell) + \ln(2\pi T_R \ell) - 2\psi(\Delta) + \frac{1}{2} \left\{ \psi\left(\frac{\Delta}{2} + i\frac{\omega}{4\pi T_L}\right) + \psi\left(\frac{\Delta}{2} - i\frac{\omega}{4\pi T_L}\right) + \psi\left(\frac{\Delta}{2} + i\frac{\omega}{4\pi T_R}\right) + \psi\left(\frac{\Delta}{2} - i\frac{\omega}{4\pi T_R}\right) \right\} \right] \sigma_{\text{abs}}^{13}. \quad (5.4)$$

Taking the low-temperature limit of $\omega \gg T_{R/L} = 1/2\pi\ell$ and $\Delta = 2$, we get the absorption cross section which is logarithmically corrected as [18]

$$\sigma_{\text{abs}}^{23}|_{\omega \gg T_{R/L}} = \pi^2 \omega \ell^2 \left(1 + 2 \ln[\omega \ell] + c_1 \right), \quad (5.5)$$

where c_1 is given by

$$c_1 = 2\gamma - 2 \ln 2 \quad (5.6)$$

with the Euler's constant $\gamma = 0.5772$. The absorption cross section (5.5) is positive definite for the low-temperature limit because $1 + 2\gamma - 2 \ln 2 \simeq 0.768$. In deriving Eq.(5.5), we have used an asymptotic form of the digamma function

$$\text{Re}[\psi(1 + ix)] = \text{Re}[\psi(1 - ix)] \simeq \ln x + \frac{1}{12x^2} + O(x^{-4}). \quad (5.7)$$

We note that Eq.(5.5) shows the absorption cross section for the logarithmic operator.

Finally, Eq.(4.10) provides the absorption cross section for the log-square operator

$$\sigma_{\text{abs}}^{33} \simeq \left\{ -\frac{3}{2(\Delta - 1)^2} - \psi'(\Delta) + \frac{1}{8}\psi'\left(\frac{\Delta}{2} + i\frac{\omega}{4\pi T_L}\right) + \frac{1}{8}\psi'\left(\frac{\Delta}{2} - i\frac{\omega}{4\pi T_L}\right) + \frac{1}{8}\psi'\left(\frac{\Delta}{2} + i\frac{\omega}{4\pi T_R}\right) + \frac{1}{8}\psi'\left(\frac{\Delta}{2} - i\frac{\omega}{4\pi T_R}\right) + \frac{1}{2}\Psi(\omega)^2 \right\} \sigma_{\text{abs}}^{13},$$

where the last term is obtained through the relation of $\sigma_{\text{abs}}^{23} \equiv \Psi(\omega)\sigma_{\text{abs}}^{13}$ defined in Eq.(5.4). We consider the low-temperature limit of $\omega \gg T_{R/L}$ and $\Delta = 2$. In this case, ψ' takes the asymptotic form

$$\text{Re}[\psi'(1 + ix)] = \text{Re}[\psi'(1 - ix)] \simeq \frac{1}{2x^2} + O(x^{-5}). \quad (5.8)$$

Then, σ_{abs}^{33} leads to the form

$$\sigma_{\text{abs}}^{33}|_{\omega \gg T_{R/L}} = \pi^2 \omega \ell^2 \left(1 + 2 \ln^2[\omega \ell] + c_2 \ln[\omega \ell] + c_3 \right), \quad (5.9)$$

where c_2 and c_3 are given by

$$\begin{aligned} c_2 &= 2(1 + 2\gamma - 2\ln 2), \\ c_3 &= -\frac{1}{6}\pi^2 + 2\gamma^2 + 2\gamma - 2(1 + 2\gamma)\ln 2 + 2(\ln 2)^2 - 1. \end{aligned} \quad (5.10)$$

Note that the absorption cross section (5.9) is positive definite for $c_2 \ln[\omega\ell] + c_3 + 1 > 0$ because $c_2 \simeq 1.536 > 0$ and $c_3 + 1 \simeq -1.849$.

We represent the whole cross sections as a three-by-three matrix

$$\sigma_{\text{abs}}^{ij} \sim \begin{pmatrix} 0 & 0 & \sigma_{\text{abs}}^{13} \\ 0 & \sigma_{\text{abs}}^{22} & \sigma_{\text{abs}}^{23} \\ \sigma_{\text{abs}}^{13} & \sigma_{\text{abs}}^{23} & \sigma_{\text{abs}}^{33} \end{pmatrix}, \quad (5.11)$$

where $\sigma_{\text{abs}}^{13} [= \sigma_{\text{abs}}^{22}]$ represents the absorption cross section when the Klein-Gordon mode is scattered by the BTZ black hole, σ_{abs}^{23} denotes the absorption cross section when the log-mode is scattered by the BTZ black hole, and σ_{abs}^{33} represents the absorption cross section when the log-square mode is scattered by the BTZ black hole. Applying the previous truncation process done for the LCFT to (5.11) leads to

$$\sigma_{\text{abs}}^{ij} \sim \begin{pmatrix} 0 & 0 \\ 0 & \sigma_{\text{abs}}^{22} \end{pmatrix}, \quad (5.12)$$

which provides the absorption cross section for the Klein-Gordon mode in the low-temperature limit.

6 Summary and conclusion

We have constructed a rank-3 finite temperature logarithmic conformal field theory (LCFT) starting from a higher-derivative scalar field model in the BTZ black hole background.

On the gravity side, this scalar field model is composed of two auxiliary scalar fields A_1 and A_2 and one relevant field A_3 . The tricritical point where the masses of the three scalars degenerate was introduced to avoid the ghost problem. It is known that the ghost issue was captured even in the Minkowski space for the non-critical case of a six-derivative scalar theory [see appendix C in the Ref.[27]]. Since the scalar A_2 satisfies the fourth-order differential equation (2.13) and A_3 satisfies the sixth-order differential equation (2.14), one

could expect that two higher-order logarithmic modes appear and these correspond to two logarithmic partners of the Klein-Gordon scalar mode. According to the AdS/CFT logic, there are two possibilities of coupling to boundary operators: the first one is symmetric and the second one is Jordan. The first provides a standard coupling of $\int d^2x [\sum_{i=1}^3 A_{i(b)} \mathcal{O}_i]$, while the second gives us the coupling of $\int d^2x [A_{3(b)} \mathcal{O}_1 + A_{1(b)} \mathcal{O}_3 + A_{2(b)} \mathcal{O}_2]$. In this work, we have chosen the Jordan which makes the coupling between boundary field $A_{i(b)}$ and operator \mathcal{O}_i less transparent. This implies that in order to obtain two-point correlator $\langle \mathcal{O}_3 \mathcal{O}_3 \rangle$, one has to vary the effective action (2.26) with respect to $A_{1(b)}$ twice, but $\langle \mathcal{O}_1 \mathcal{O}_1 \rangle$ with respect to $A_{3(b)}$ twice. This could be also observed from (2.5) obtained when varying (2.1) with respect to Φ_1 but not Φ_3 .

On the CFT side, all correlators describe a rank-three finite temperature LCFT. Its zero temperature limit reduces to a rank-3 LCFT which is dual of a higher-derivative scalar field model in the AdS_3 background [10]. Here, a truncation allowed the theory to have a unitary subspace by throwing away all modes which generate the third column and row of (3.9).

We have computed the retarded real-time Green's functions and retarded Green's functions in the momentum space to know the locations of poles. Then, the AdS/LCFT correspondence implies that quasinormal frequencies determining the relaxation times of perturbations around the black hole are agreed with the location of poles of the retarded Green's function of the corresponding perturbations in the dual CFT. For a tricritical GMG, we read off the log-square quasinormal frequencies of graviton from the poles of the retarded Green's function in the momentum space. This confirms the previously approximate computation [16]. After implementing the truncation process, we have found quasinormal frequencies from a unitary conformal field theory.

Finally, we have obtained the absorption cross sections by using the retarded Green's functions on the boundary. Actually, it is not an easy task to derive all absorption cross sections because two scalars A_2 and A_3 satisfy the higher-order differential equations (2.13) and (2.14). However, the boundary retarded Green's functions allow us to compute the absorption cross sections easily. In the low-temperature limit of $\omega \gg T_{R/L}$, one has $\sigma_{\text{abs}}^{13} = \sigma_{\text{abs}}^{22} = \pi^2 \omega \ell^2$, $\sigma_{\text{abs}}^{23} = \pi^2 \omega \ell^2 (1 + 2 \ln[\omega \ell] + c_1)$, and $\sigma_{\text{abs}}^{33} = \pi^2 \omega \ell^2 (1 + 2 \ln^2[\omega \ell] + c_2 \ln[\omega \ell] + c_3)$. We note that $\ln[\omega \ell]$ and $\omega \ell \ln^2[\omega \ell]$ are signals to indicate the higher-order differential equations for A_2 and A_3 in the bulk. Imposing the truncation on the absorption cross

sections, one has $\sigma_{\text{abs}}^{22} = \pi^2 \omega \ell^2$ obtained from scattering the Klein-Gordon mode off the BTZ black hole.

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